Constitutive model for the fluid-particle drag coefficient in filtered two-fluid models for gas-particle flows

Chris Milioli, Fernando Milioli, William Holloway,
Kapil Agrawal & Sankaran Sundaresan
Princeton University

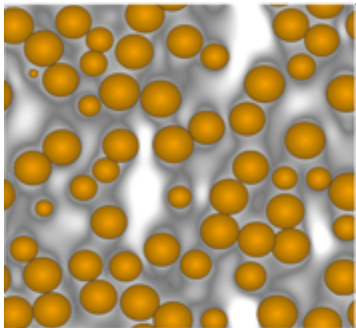
Paper 54b, 03B01 Special session to celebrate Tom O'Brien's career long accomplishments
Monday, October 29, 2012
Conference C (Omni)

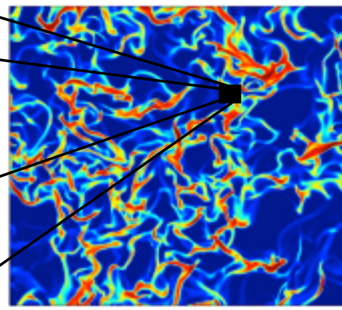
Filtered two-fluid model: Overview

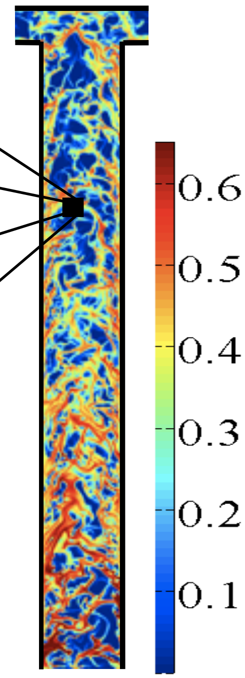
MICRO-SCALE~ 50*μm-mm*

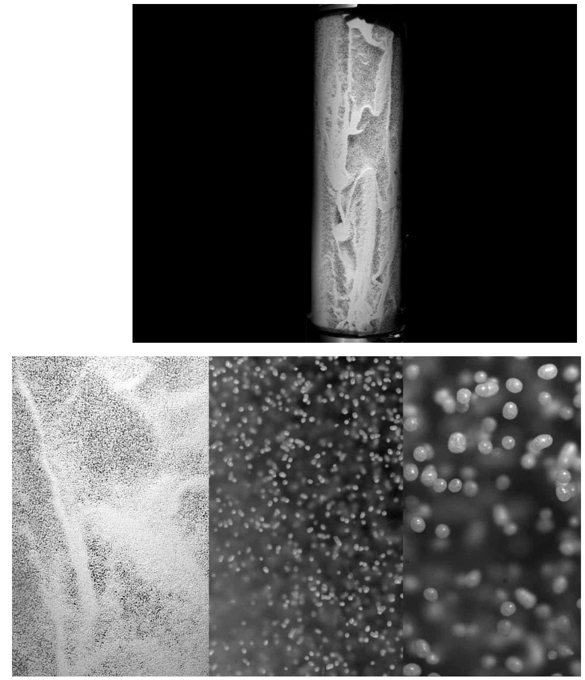
MESO-SCALE ~ *mm-cm*

MACRO-SCALE ~ *cm-m*









DEM for particles, DNS or CFD of averaged equations for the fluid flow.

Volume-averaged hydrodynamic models for fluid and particle phases.

Filtered volume-averaged hydrodynamic models for fluid and particle phases.

High Speed Particle Imaging: Riser Flow

Engineering need: tools to probe macro-scale flow features directly.

Courtesy: Franklin Shaffer, NETL, Morgantown, WV (2009)

Filtered two-fluid model: Overview

Develop models that allow us to focus on large-scale flow structures, without ignoring the possible consequence of the smaller scale structures.

Original two-fluid model and constitutive relations

Significant advances in the past three decades

→

Filtered two-fluid model

Modified constitutive relations for

hydrodynamic terms

*species and energy dispersion**

*interphase heat and mass transfer rates**

even modified reaction rate expressions!

Agrawal et al. (2001); Andrews et al., (2005); Igci et al., (**2008**, 2010, 2011a, 2011b)

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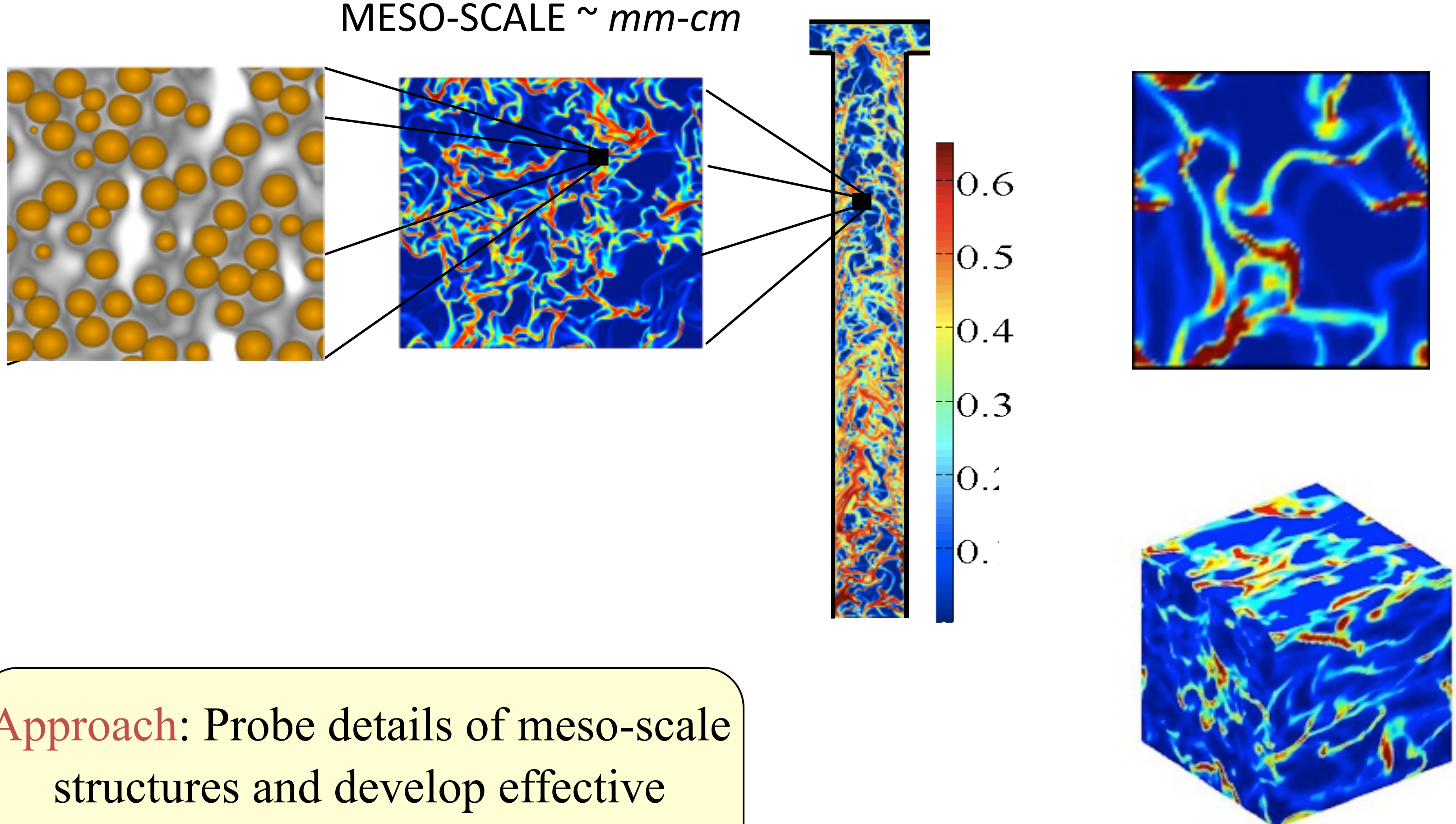


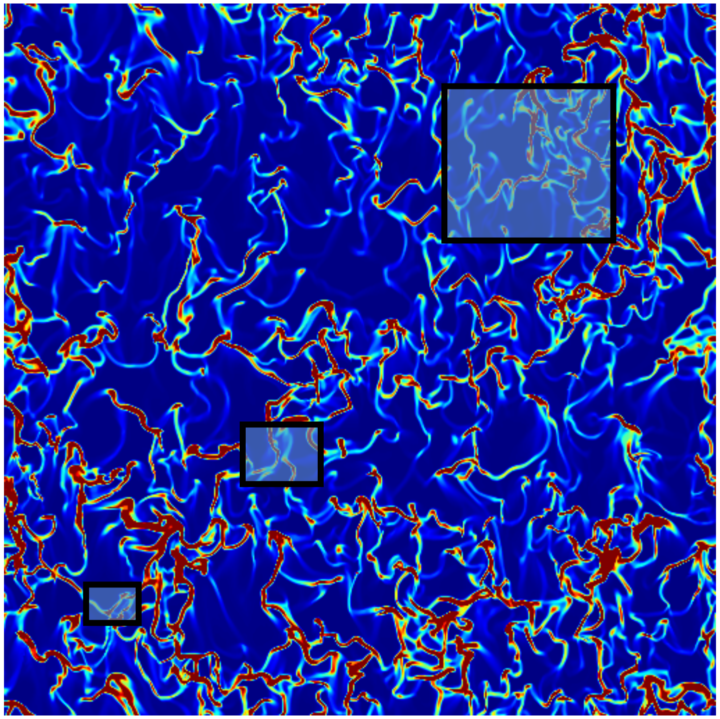
Approach: Probe details of meso-scale structures and develop effective coarse-grained equations

Filtered two-fluid model: Overview

Filter "data" generated through highly resolved simulations of two-fluid models

- Snapshot of particle volume fraction field – kinetic theory based two-fluid model.
- Squares of different sizes illustrate regions (i.e. filters) of different sizes.

$$\tilde{\mathbf{v}}_g = \frac{\overline{\phi_g \mathbf{v}_g}}{\overline{\phi}_g}; \qquad \tilde{\mathbf{v}}_s = \frac{\overline{\phi_s \mathbf{v}_s}}{\overline{\phi}_s}$$



$$\Delta_{grid} << \Delta_{fil} << \Delta_{domain}$$

Igci et al., (2008)

Filtered drag coefficient

Filter "data" generated through highly resolved simulations of two-fluid models

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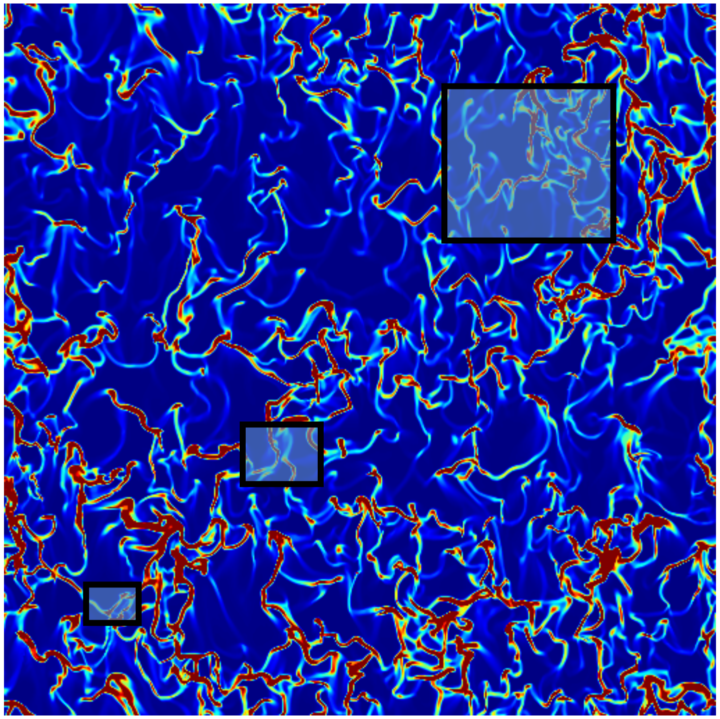
$$\beta_{fil} = \text{filtered drag coefficient}$$

$$= \frac{\overline{\beta_{micro}\left(\phi_s, \left|\mathbf{v}_g - \mathbf{v}_s\right|\right)\left(\mathrm{v}_{gy} - \mathrm{v}_{sy}\right) - \phi_s' \frac{\partial p_g'}{\partial y}}}{\left(\tilde{\mathrm{v}}_{gy} - \tilde{\mathrm{v}}_{sy}\right)}$$



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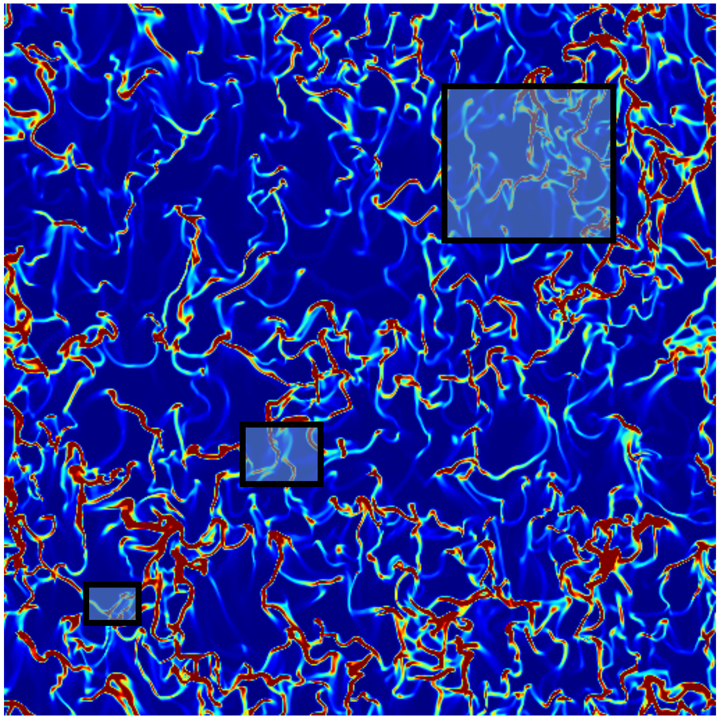
$$= \beta_{micro}\left(\overline{\phi}_s, \left|\tilde{\mathbf{v}}_g - \tilde{\mathbf{v}}_s\right|\right)(1 - H)$$

$$H = f\left(\frac{g\Delta_{fil}}{v_t^2}, \underbrace{\cdots\cdots\cdots\cdots}_{\substack{\text{parameters characterizing} \\ \text{sub-filter scale structure}}}\right)$$



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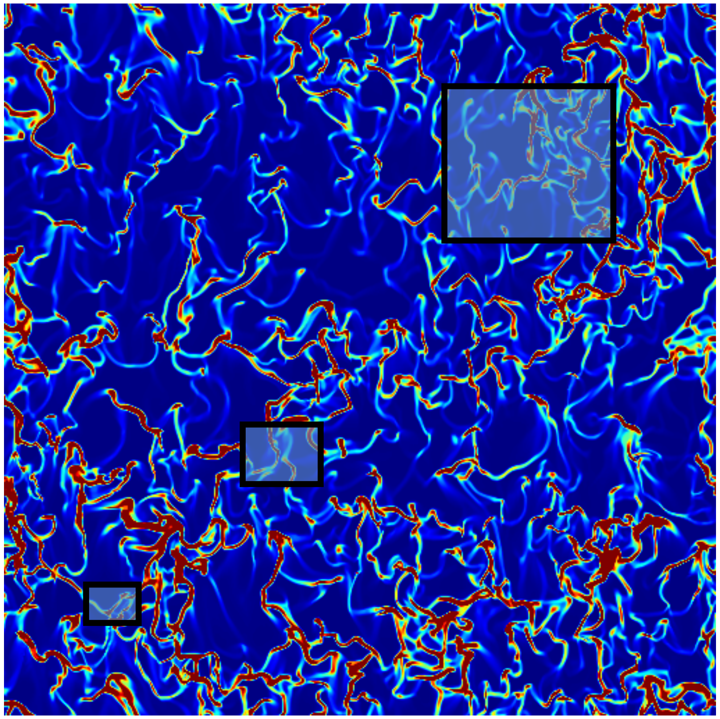

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Igci et al., (**2008**, **2010, 2011a, 2011b**)

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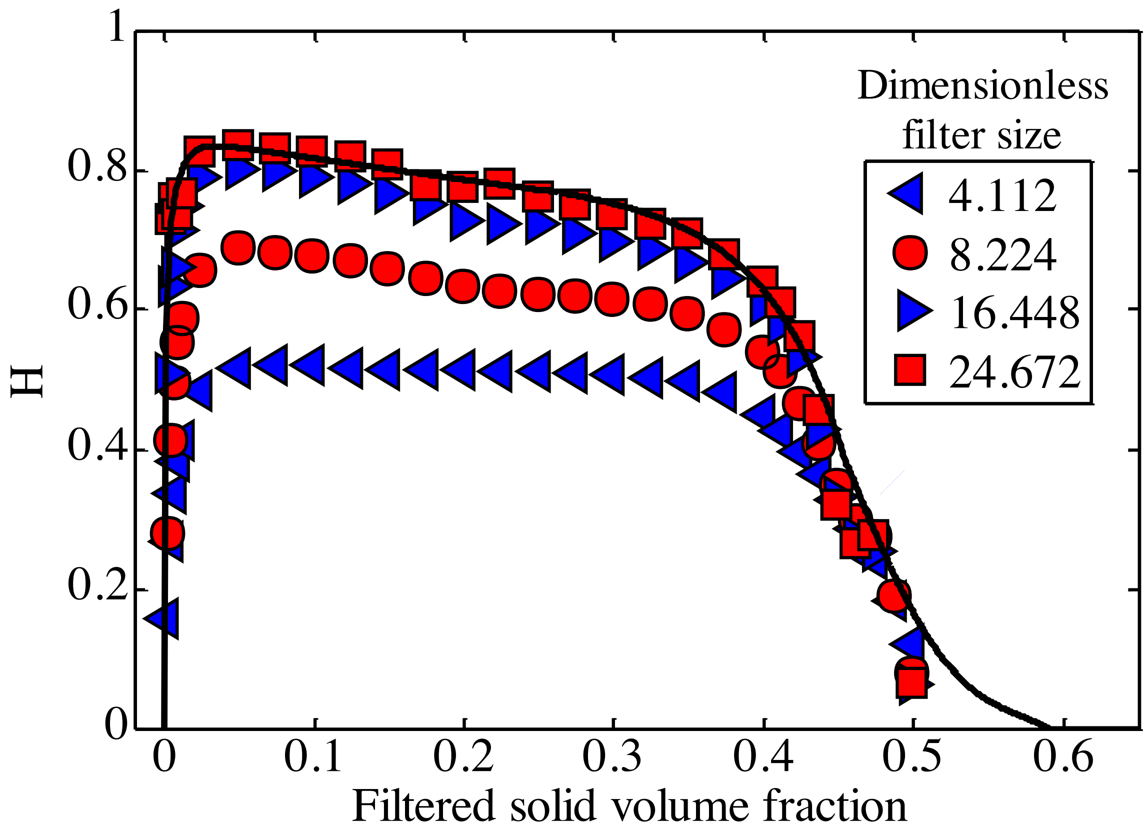
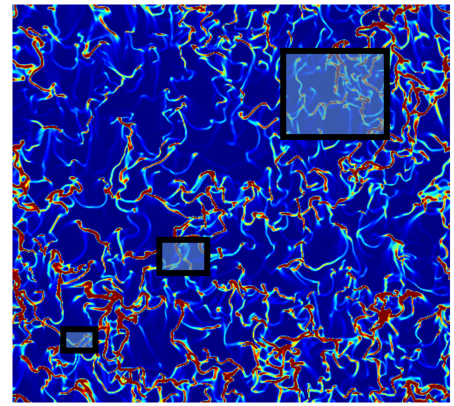




$$\begin{Bmatrix} 75\mu m \text{ FCC particles} \\ \text{ambient air} \end{Bmatrix}$$

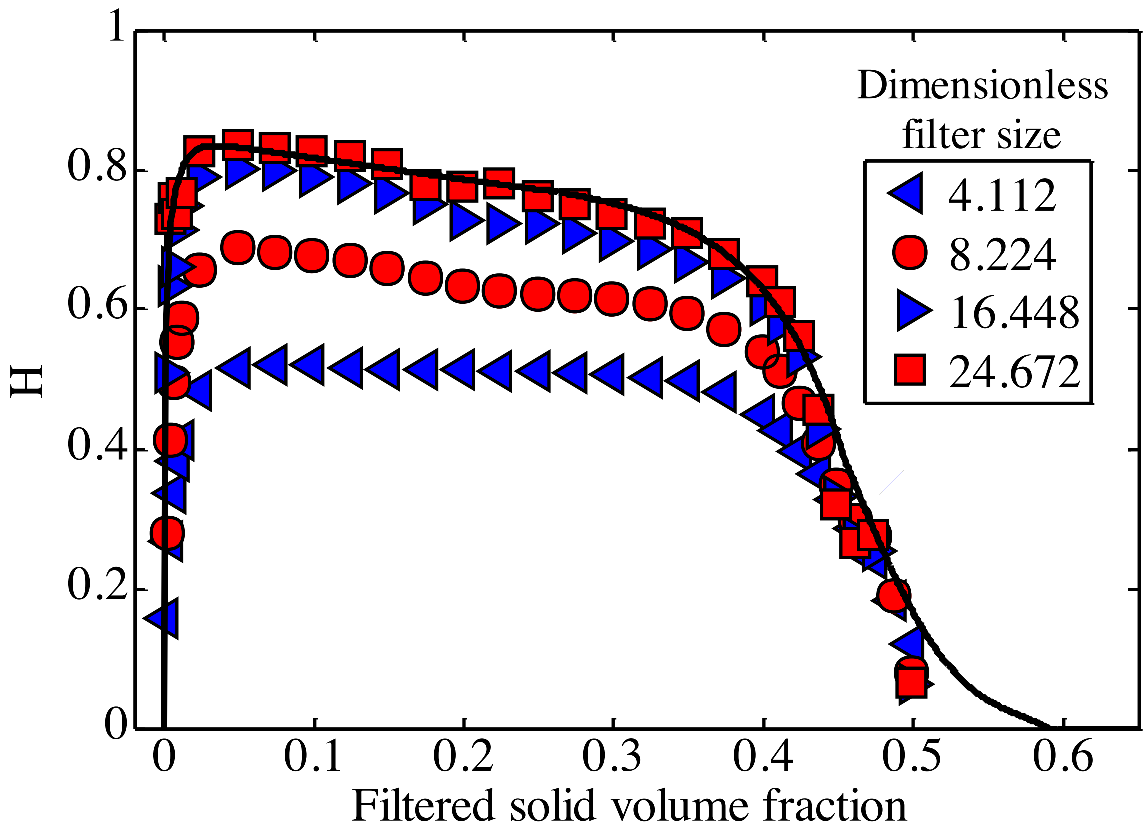
$$\frac{g\Delta_{fil}}{v_t^2} = 2.056 \Rightarrow \Delta_{fil} = 1cm$$

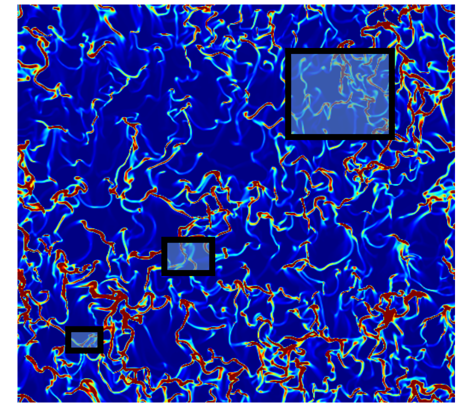
$$\beta_{fil} = \beta_{micro}\left(\bar{\phi}_s, \left|\tilde{\mathbf{v}}_g - \tilde{\mathbf{v}}_s\right|\right)(1 - H);$$

$$H = f\left(\frac{g\Delta_{fil}}{v_t^2}, \bar{\phi}_s\right)$$

Igci et al., (2008, **2010, 2011a, 2011b**)

Filtered drag coefficient: The present study





$$\left\{ \begin{array}{l} 75\mu m \text{ FCC particles} \\ \text{ambient air} \end{array} \right\}$$

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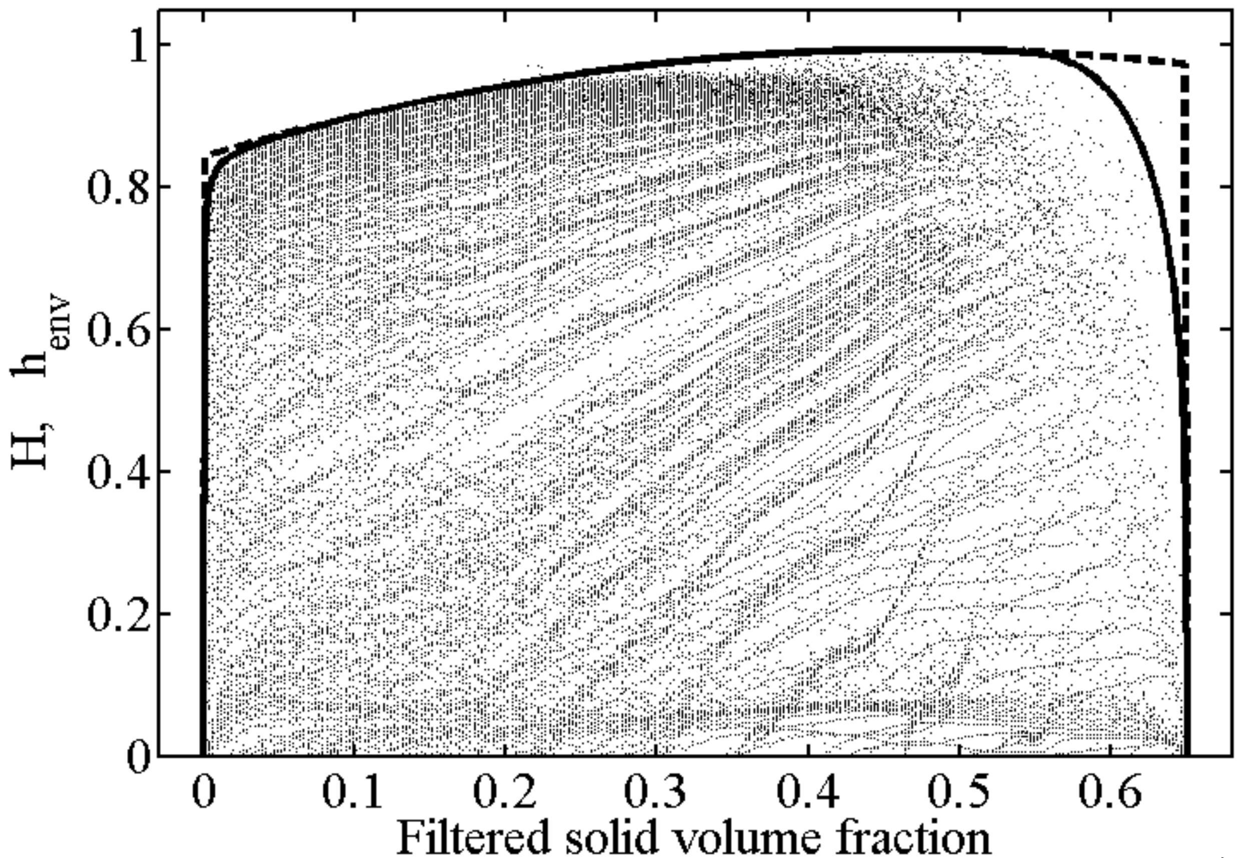

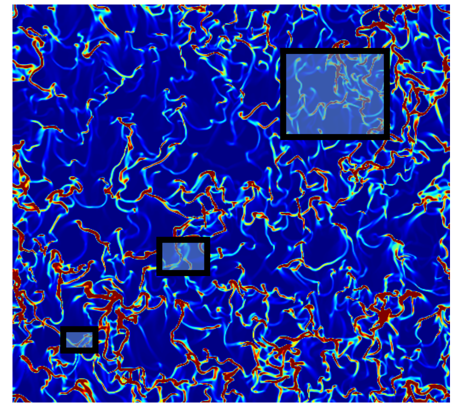

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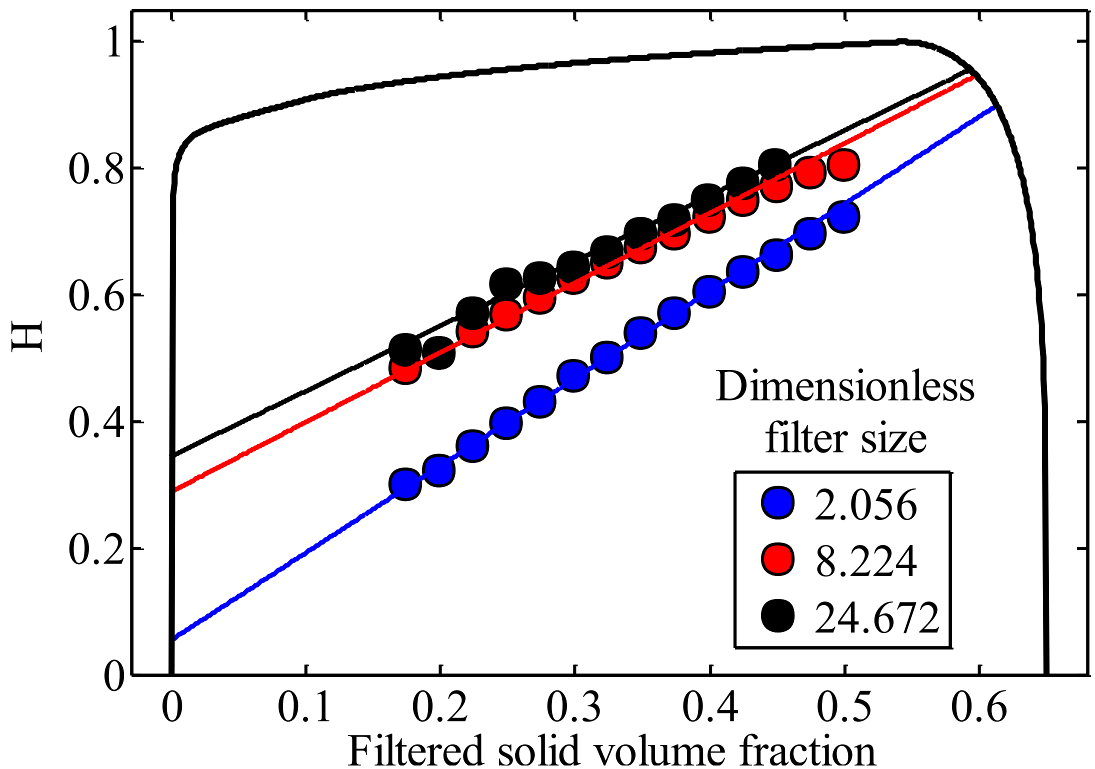
Filtered drag coefficient: The present study

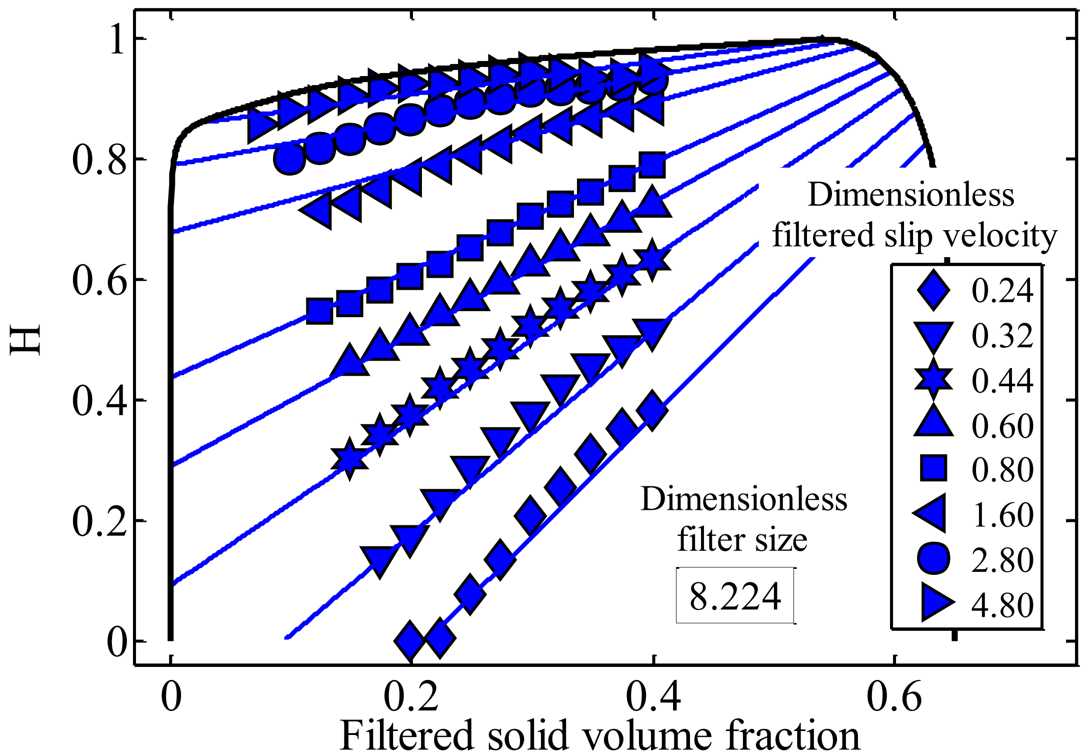


$$\frac{\langle|\widetilde{v}_{slip}|\rangle}{v_t} = 0.60$$

- As filter size increases, the filtered drag coefficient decreases.
- Does suggest the existence of large filter size asymptote.

$$H = f\left(\frac{g\Delta_{fil}}{v_t^2}, \bar{\phi}_s, \frac{|\tilde{\mathbf{v}}_g - \tilde{\mathbf{v}}_s|}{v_t}\right) \quad \left\{\begin{array}{l}75\mu m \text{ FCC particles} \\ \text{ambient air}\end{array}\right\} \quad \frac{g\Delta_{fil}}{v_t^2} = 2.056 \Rightarrow \Delta_{fil} = 1cm$$

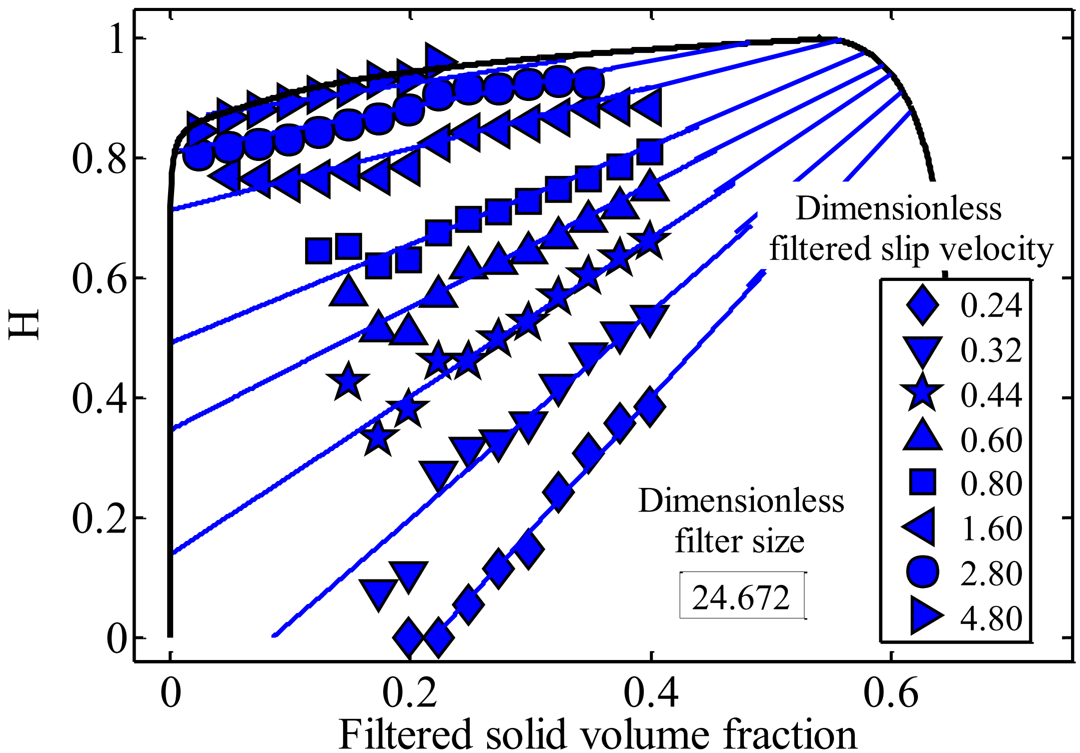
Filtered drag coefficient: The present study



- As slip velocity increases, the filtered drag coefficient decreases.

$$H = f\left(\frac{g\Delta_{fil}}{v_t^2}, \bar{\phi}_s, \frac{\left|\tilde{\mathbf{v}}_g - \tilde{\mathbf{v}}_s\right|}{v_t}\right) \quad \left\{\begin{array}{l} 75\mu m \text{ FCC particles} \\ \text{ambient air} \end{array}\right\} \quad \frac{g\Delta_{fil}}{v_t^2} = 2.056 \Rightarrow \Delta_{fil} = 1cm$$

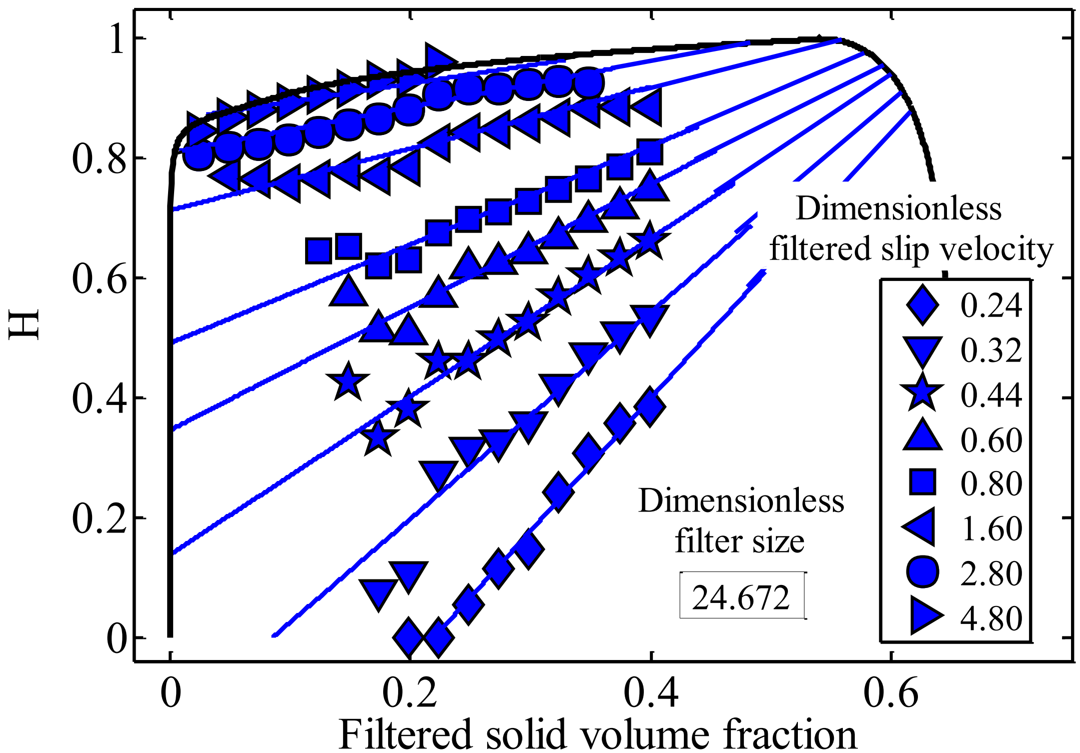
Filtered drag coefficient: The present study



- As slip velocity increases, the filtered drag coefficient decreases.
- Same trend at different filter sizes.

$$H = f\left(\frac{g\Delta_{fil}}{v_t^2}, \bar{\phi}_s, \frac{\left| \tilde{\mathbf{v}}_g - \tilde{\mathbf{v}}_s \right|}{v_t} \right) \quad \left\{ \begin{array}{l} 75\mu m \text{ FCC particles} \\ \text{ambient air} \end{array} \right\} \quad \frac{g\Delta_{fil}}{v_t^2} = 2.056 \Rightarrow \Delta_{fil} = 1cm$$

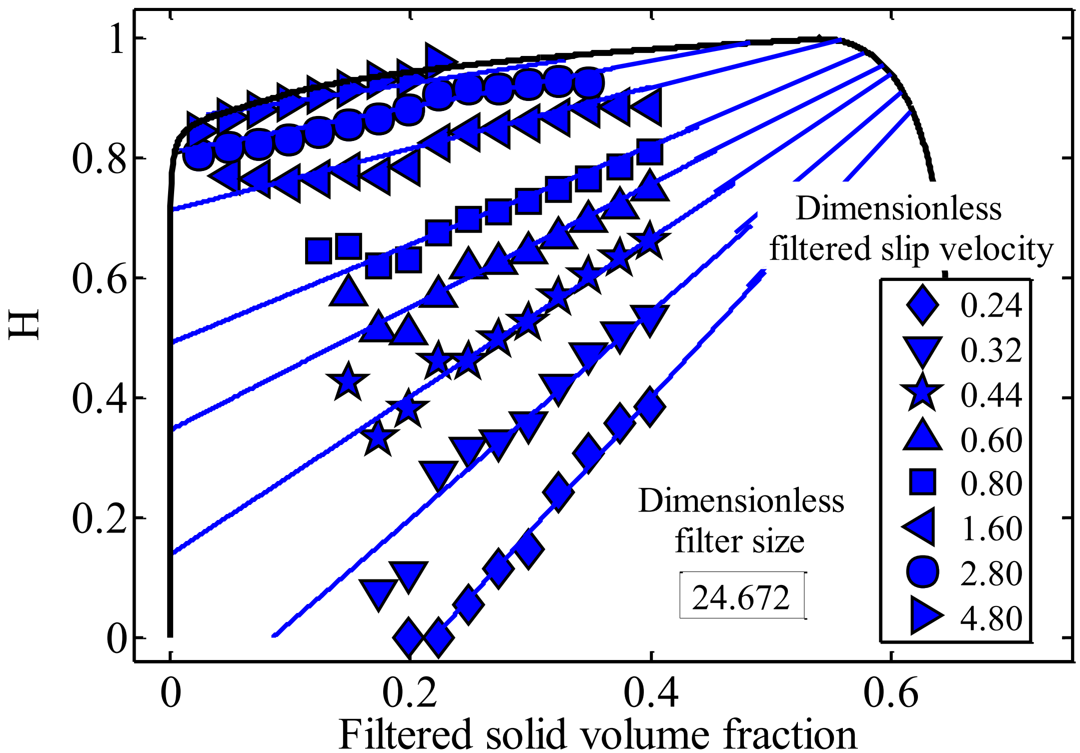
Filtered drag coefficient: The present study

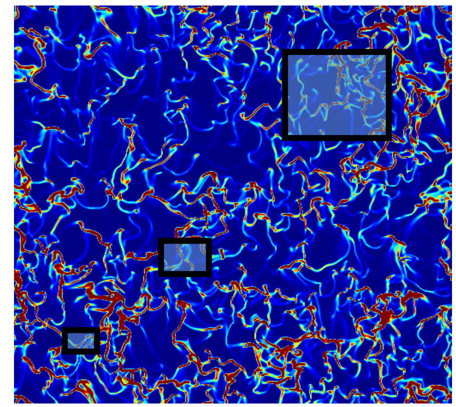


- As slip velocity increases, the filtered drag coefficient decreases.
- NOT THE USUAL INERTIAL CORRECTION!

$$H = f\left(\frac{g\Delta_{fil}}{v_t^2}, \bar{\phi}_s, \frac{\left|\tilde{\mathbf{v}}_g - \tilde{\mathbf{v}}_s\right|}{v_t}\right) \quad \begin{Bmatrix} 75\mu m \text{ FCC particles} \\ \text{ambient air} \end{Bmatrix} \quad \frac{g\Delta_{fil}}{v_t^2} = 2.056 \Rightarrow \Delta_{fil} = 1cm$$

Filtered drag coefficient: The present study

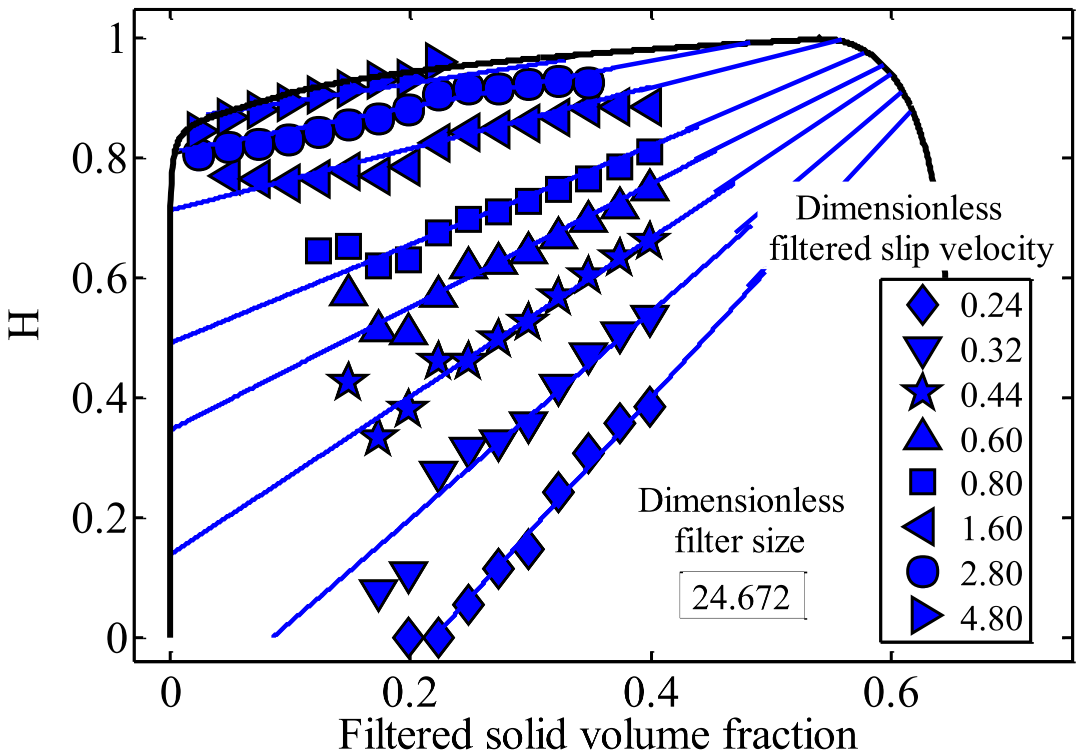


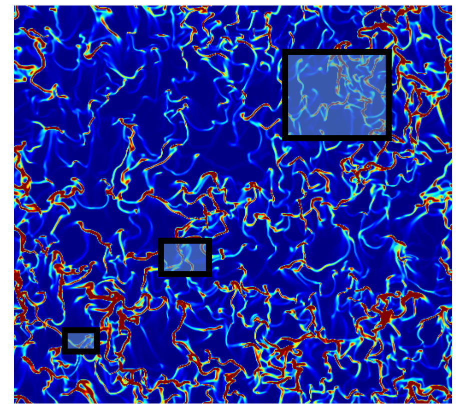


As the slip velocity increases, the sub-filter scale distribution of particles becomes more segregated

$$H = f\left(\frac{g\Delta_{fil}}{v_t^2}, \bar{\phi}_s, \frac{|\tilde{\mathbf{v}}_g - \tilde{\mathbf{v}}_s|}{v_t}\right) \quad \begin{Bmatrix} 75\mu m \text{ FCC particles} \\ \text{ambient air} \end{Bmatrix} \quad \frac{g\Delta_{fil}}{v_t^2} = 2.056 \Rightarrow \Delta_{fil} = 1cm$$

Filtered drag coefficient: The present study





At low slip velocities:

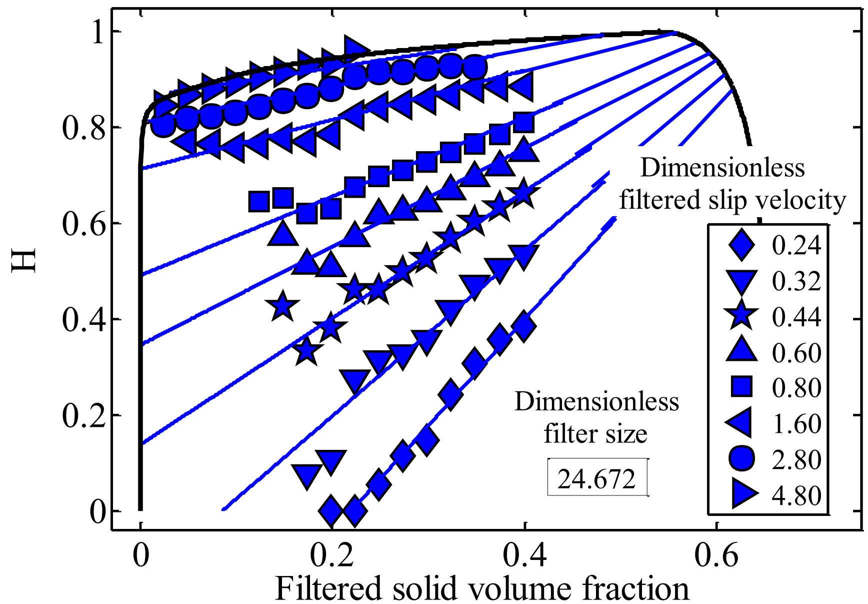
$$H_1 = \begin{cases} 0, & \overline{\phi_s} < \overline{\phi_s^c} \\ A\left(\overline{\phi_s} - \overline{\phi_s^c}\right), & \overline{\phi_s} > \overline{\phi_s^c} \end{cases}$$

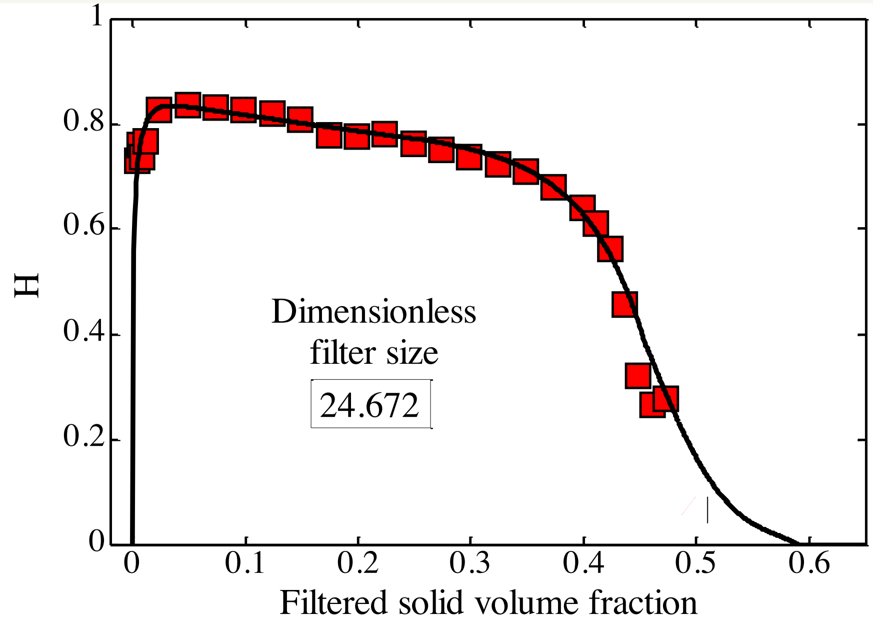
At high slip velocities:

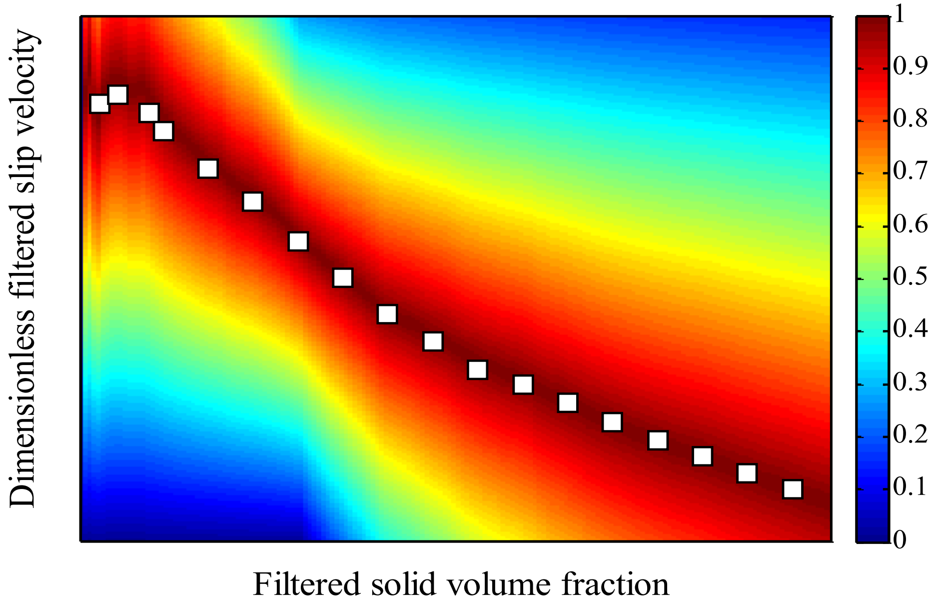
$$H_1 = B + A\overline{\phi_s}$$

$$H = \min\left(h_{env},\, \mathrm{H}_1\right)$$

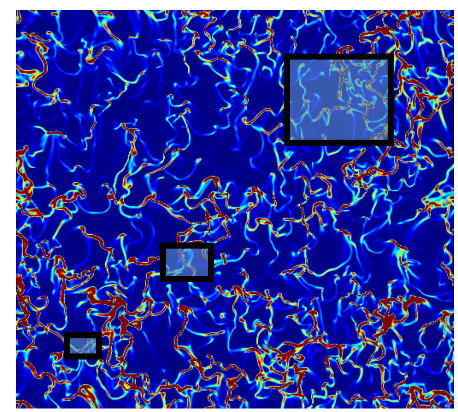
Filtered drag coefficient: The present study





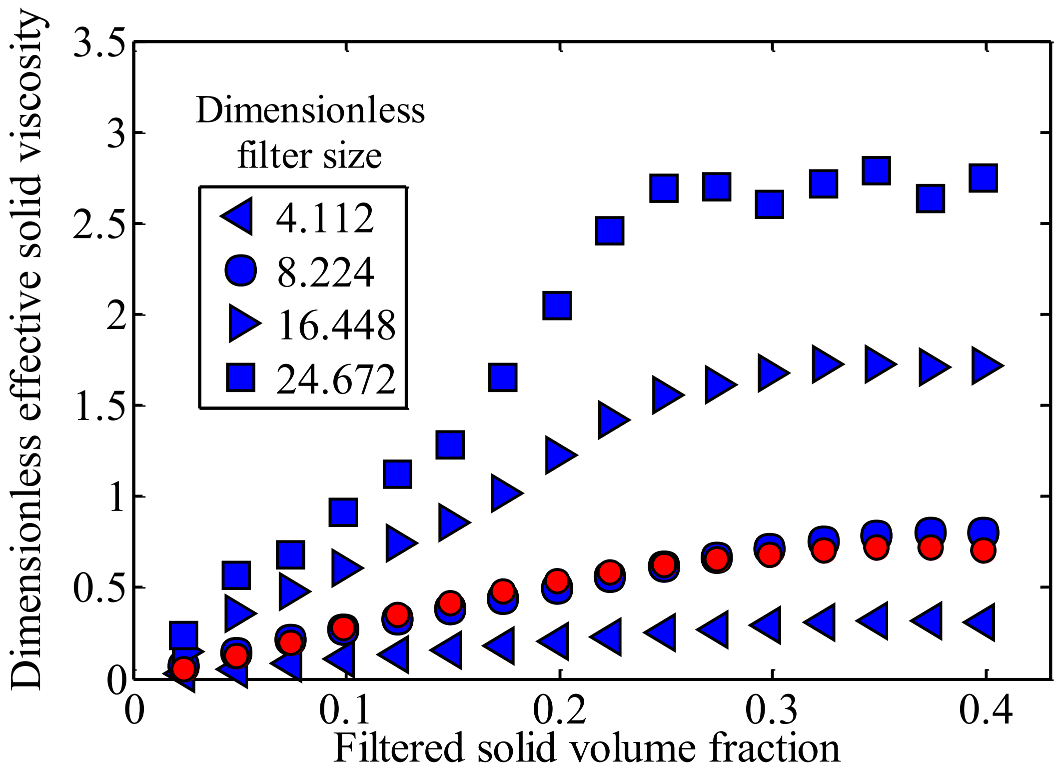


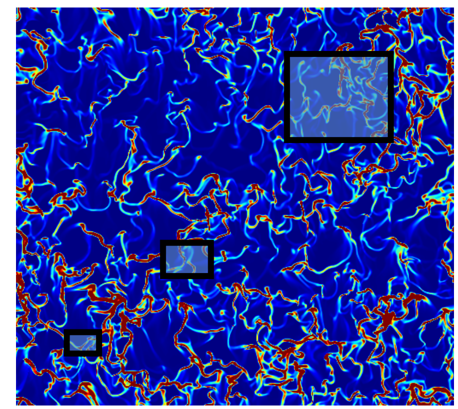
$$H = f\left(\frac{g\Delta_{fil}}{v_t^2}, \bar{\phi}_s, \frac{\left| \tilde{\mathbf{v}}_g - \tilde{\mathbf{v}}_s \right|}{v_t} \right)$$



Filtered particle phase viscosity

$$\frac{\mu_s g}{\rho_s v_t^3}$$





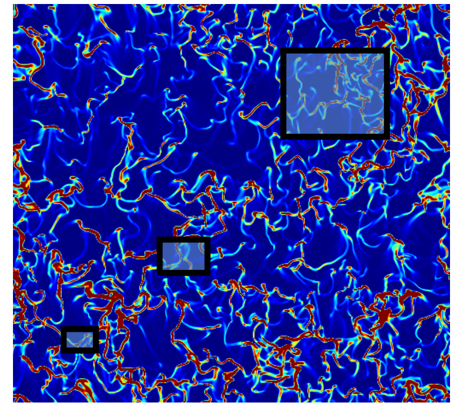
$$\frac{\mu_s g}{\rho_s v_t^3} = f\left(\frac{g\Delta_{fil}}{v_t^2}, \bar{\phi}_s\right) \quad \begin{Bmatrix} 75\mu m \text{ FCC particles} \\ \text{ambient air} \end{Bmatrix} \quad \frac{g\Delta_{fil}}{v_t^2} = 4.112 \Rightarrow \Delta_{fil} = 2cm$$

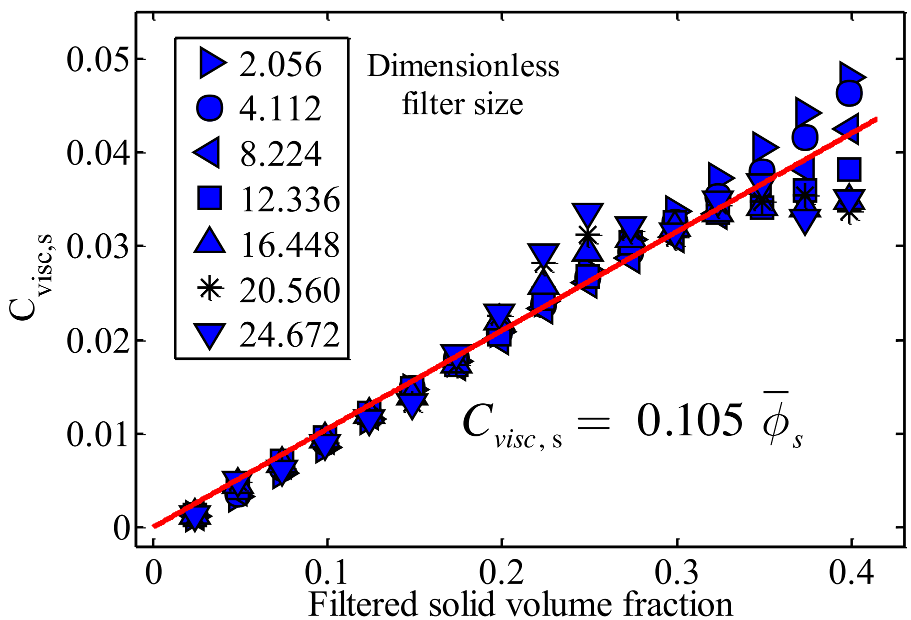
Igci et al., (2008, **2010, 2011a, 2011b**)

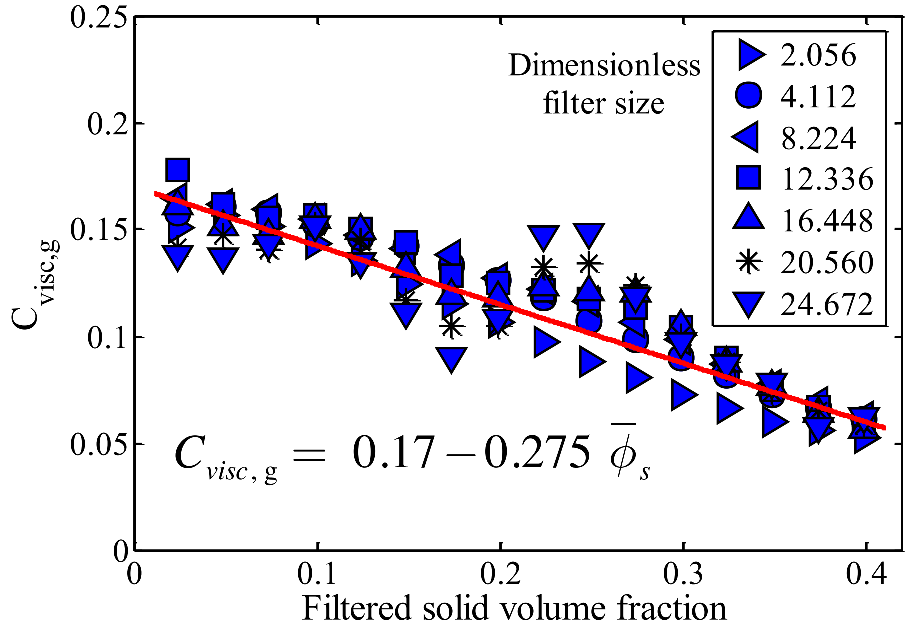
Filtered particle phase viscosity: Present study

$$\bar{S}_i = \sqrt{2\bar{\mathbf{S}}_i : \bar{\mathbf{S}}_i}, \quad \bar{\mathbf{S}}_i = \frac{1}{2}\left(\nabla\tilde{\mathbf{v}}_i + \nabla\tilde{\mathbf{v}}_i^T\right) - \frac{1}{3}\left(\nabla\cdot\tilde{\mathbf{v}}\right)\mathbf{I}, \quad i = s,g$$

$$\mu_{fil,i} = \rho_i \Delta_{fil}^2 \bar{S}_i C_{visc,i}, \quad i = s,g$$







Summary

- A more refined model for the filtered fluid-particle drag force is presented.

- Smagorinsky-like model for the filtered particle and fluid phase viscosities capture the computationally generated data nicely.

- Smagorinsky-like model for the meso-scale particle and fluid phase pressures (akin to turbulent kinetic energy) works nicely as well (not presented).

Acknowledgments

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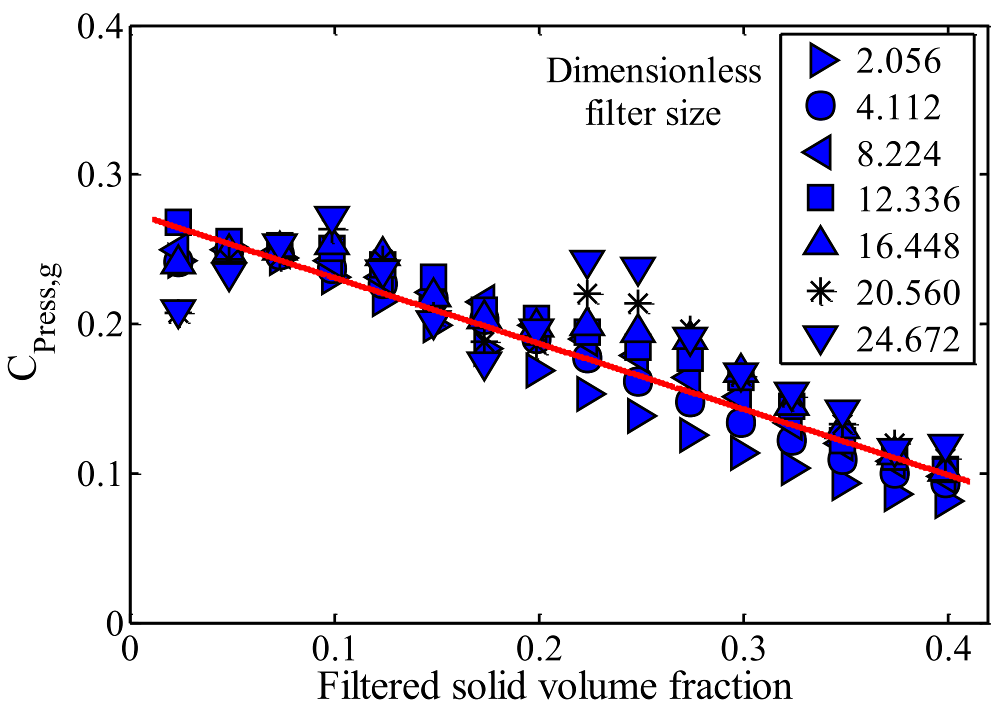
- FAPESP - São Paulo State Research Foundation (Brazil)
  Department of Energy CCSI
- ExxonMobil Research & Engineering Co.

Disclaimer

Filtered meso-scale pressure: Present study

$$\bar{S}_i = \sqrt{2\bar{\mathbf{S}}_i : \bar{\mathbf{S}}_i}, \quad \bar{\mathbf{S}}_i = \frac{1}{2}\left(\nabla\tilde{\mathbf{v}}_i + \nabla\tilde{\mathbf{v}}_i^T\right) - \frac{1}{3}\left(\nabla\cdot\tilde{\mathbf{v}}\right)\boldsymbol{I}, \quad i = s, g$$

$$P_{fil,i} = \rho_i \Delta_{fil}^2 \bar{S}_i^2 \left(\frac{g\Delta_{fil}}{v_t^2}\right)^{2/7} C_{press,i}, \quad i = s, g$$



$$C_{press,g} = 0.275 - 0.44\,\bar{\phi}_s$$

Filtered meso-scale pressure: Present study

$$\bar{S}_i = \sqrt{2\bar{\mathbf{S}}_i : \bar{\mathbf{S}}_i}, \quad \bar{\mathbf{S}}_i = \frac{1}{2}\left(\nabla\tilde{\mathbf{v}}_i + \nabla\tilde{\mathbf{v}}_i^T\right) - \frac{1}{3}\left(\nabla\cdot\tilde{\mathbf{v}}\right)\boldsymbol{I}, \quad i = s,g$$

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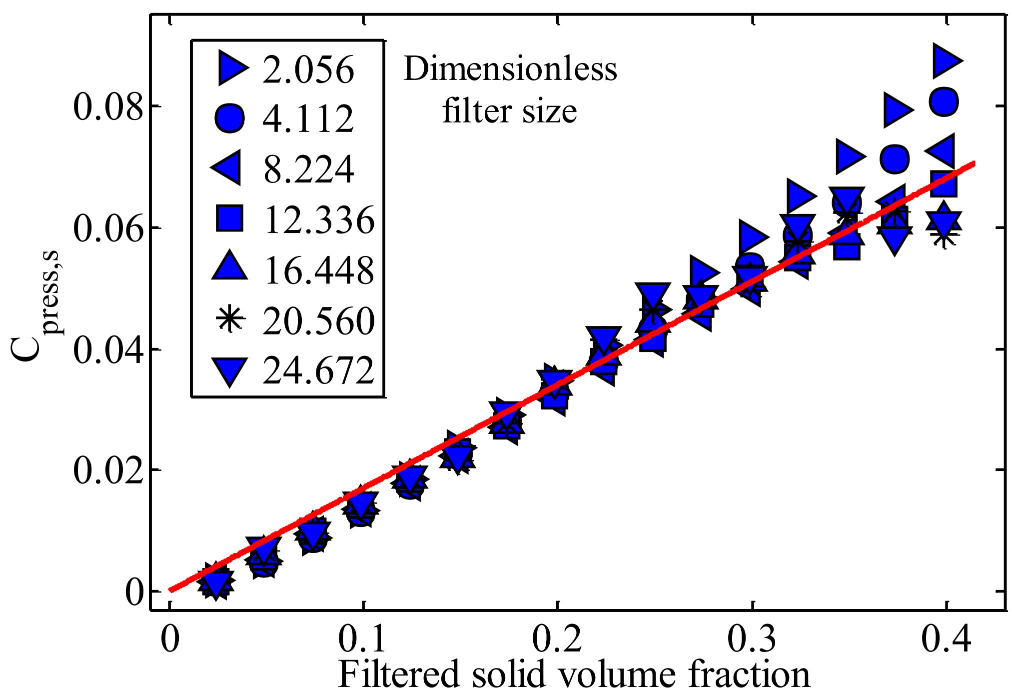

$$C_{press,s} = 0.17\ \bar{\phi}_s$$